Benha University Faculty of Engineering – Shoubra Department of Industrial Engineering Course: Mathematics 1 Code: EMP 101		Final Exam Date: May 20 , 2017 Duration Time : 2 hours Answer **All** questions

- The exam consists of one page
- No. of questions: 4 Total Mark: 40

Question 1

(a)Find **y`** from the following: 6

(i) $y = 2x^4 - 4^x + x$ (ii) $y = e^x . \sin x$ (iii) $y = \log x + \ln x$

(iv) $y = 3 + 2x + \frac{1}{x^3}$ (v) $y = \cos x . \ln x$ (vi) $y = (x + \tan x)^8$

(b) Determine the maximum and minimum points of : $f(x) = x^3 - 3x^2 + 1$ 3

(c) Write the Maclurin's expansion of : $f(x) = x + 2\cos x$ 3

Question 2

Find the integrals: 6

(a) $\int (x^3 + 3^x + 3)\, dx$ (b)$\int (3 - x^2)^2\, dx$ (c) $\int (x + x^{-3} + e^x)\, dx$

(d) $\int (2x + \cos x)\ dx$ (e) $\int (\frac{2}{3} + \frac{1}{x^3})\, dx$ (f)$\int (x - 2\sin x)\, dx$

Question 3

(a) If $A = \begin{bmatrix} 1 & -3 & 2 \\ 2 & -2 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 3 & 0 & 4 \\ 1 & 2 & 3 \end{bmatrix}$ and $C = \begin{bmatrix} 2 & -3 & 3 \\ 0 & 1 & 2 \\ 1 & -1 & 0 \end{bmatrix}$ 8

Find, if possible, A + B, A + C, A.B , A^t , |A| , |C| and C^{-1} .

(b) Find the eigenvalues and the eigenvectors of the matrix $A = \begin{bmatrix} 2 & 3 \\ 2 & 1 \end{bmatrix}$. 4

Question 4

(a)State the types of solution of a linear system. 2

(b)Write the linear system in matrix form and solve it : 3

$x + y + z = 6,\ \ -x + z + 3y = 8,\ \ y + 3z = 11.$

(c)Using the binomial theorem, expand $\frac{1}{\sqrt{1+2x}}$ 2

(d)If $z_1 = 2 + 3i$, $z_2 = 1 - 2i$. Find $z_1 + z_2$, $z_1 . z_2$, z_1 / z_2 . 3

Good Luck *Dr. Mohamed Eid*

Model Answer

Answer of Question 1

(a)(i) $y` = 8x^3 - 4^x.\ln 4 + 1$ (ii) $y` = e^x.\cos x + e^x.\sin x$

(iii) $y` = \frac{1}{\ln 10}.\frac{1}{x} + \frac{1}{x}$ (iv) $y` = 0 + 2 - 3x^{-4}$

(v) $y` = \cos x.\frac{1}{x} - \sin x.\ln x$ (vi) $y` = 8(x + \tan x)^7.(1 + \sec^2 x)$

---*6 Marks*

(b) Since $f`(x) = 3x^2 - 6x = 0$. Then x = 0, x = 2.

From $f``(x) = 6x - 6$.

We see that $f``(2) = 12 - 6 = 6$, then x = 2 is minimum.

We see that $f``(0) = 0 - 6 = -6$, then x = 0 is maximum.

---*3 Marks*

(c) Since $f(0) = 0 + 2.\cos 0 = 2$

$f`(x) = 1 - 2\sin x$, then $f`(0) = 1 - 2\sin 0 = 1$

$f``(x) = 0 - 2\cos x$, then $f``(0) = -2\cos 0 = -2$

Then $f(x) = 2 + x + \frac{-2}{2!}x^2 + \cdots$

---*3 Marks*

Answer of Question 2

(a) $I = \frac{1}{4}x^4 + \frac{1}{\ln 3}3^x + 3x + c$

(b) $I = \int(9 - 6x^2 + x^4)\,dx = 9x - 2x^3 + \frac{1}{5}x^5 + c$

(c) $I = \frac{1}{2}x^2 + \frac{x^{-2}}{-2} + e^x + c$ (d) $I = x^2 + \sin x + c$

(e) $I = \frac{2}{3}x - \frac{1}{2}x^{-2} + c$ (f) $I = \frac{1}{2}x^2 + 2\cos x + c$

---*6 Marks*

Answer of Question 3

(a) A + C, A.B, |A| are impossible.

$$A + B = \begin{bmatrix} 4 & -3 & 6 \\ 3 & 0 & 4 \end{bmatrix}, \quad A` = \begin{bmatrix} 1 & 2 \\ -3 & -2 \\ 2 & 1 \end{bmatrix}, \quad |C| = -5 \quad \text{and} \quad C^{-1} = \frac{1}{5}\begin{bmatrix} -2 & 3 & 9 \\ -2 & 3 & 4 \\ 1 & 1 & -2 \end{bmatrix}$$

--*8 Marks*

(b) $\begin{vmatrix} 2-\lambda & 3 \\ 2 & 1-\lambda \end{vmatrix} = (2-\lambda)(1-\lambda) - 6 = \lambda^2 - 3\lambda - 4 = 0$. Then $\lambda_1 = -1, \quad \lambda_2 = 4$

From the equation, $\begin{bmatrix} 2-\lambda & 3 \\ 2 & 1-\lambda \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = 0$

For : $\lambda_1 = -1$, $\begin{bmatrix} 3 & 3 \\ 2 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = 0$

Then 3x + 3y = 0, 2x + 2y = 0

Then x = – y = any number except 0

Put y = 1, we get x = – 1 and the

corresponding eigenvector is: $X_1 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$

For : $\lambda_2 = 4$, $\begin{bmatrix} -2 & 3 \\ 2 & -3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = 0$

Then –2x + 3y = 0, 2x – 3y = 0

Then 2x = 3y = any number except 0

Put y = 2, we get x = 3 and the

corresponding eigenvector is: $X_2 = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$

--*4 Marks*

Answer of Question 4

(a)Types of solutions : one solution, infinite number of solutions, no solution.

--*2 Marks*

(b) $\begin{bmatrix} 1 & 1 & 1 \\ -1 & 3 & 1 \\ 0 & 1 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 8 \\ 11 \end{bmatrix}$. The solution is (1, 2, 3).

--*3 Marks*

(c) $\frac{1}{\sqrt{1+2x}} = (1+2x)^{-\frac{1}{2}} = 1 - x + \frac{3}{2}x^2 + \cdots, \quad |2x| < 1$

--*2 Marks*

(d) $z_1 + z_2 = 3 + i, \quad z_1.z_2 = 8 - i, \quad \frac{z_1}{z_2} = -\frac{4}{5} + \frac{7}{5}i$

--*3 Marks*

Dr. Mohamed Eid